



TITLE:

Numerical Study of the Formation of Star Dunes with Three and Four arms (Mathematical Aspects of Pattern Formation in Complex Fluids)

AUTHOR(S):

Zhang, Ruyan; Kawamura, Tetuya

CITATION:

Zhang, Ruyan ...[et al]. Numerical Study of the Formation of Star Dunes with Three and Four arms (Mathematical Aspects of Pattern Formation in Complex Fluids). 数理解析研究所講究録 2005, 1413: 1-8

ISSUE DATE:

2005-02

URL:

<http://hdl.handle.net/2433/26194>

RIGHT:

Numerical Study of the Formation of Star Dunes With Three and Four arms

鳥取大学乾燥地研究センター 張 汝岩 (Ruyan ZHANG)

Arid Land Research Center, Tottori University

お茶の水女子大学人間文化研究科 河村哲也 (Tetuya KAWAMURA)

Graduate School of Humanities and Sciences, Ochanomizu University

Two kinds of Star dunes are simulated numerically in order to make clear the mechanism of their formation. The flow above the sand dune has been investigated by using Large-Eddy Simulation (LES) method. The numerical method employed in this study can be divided in three parts: (i) Calculation of the air flow above the sand dune by using standard MAC method with a generalized coordinate system; (ii) Estimation of the sand transfer caused by the flow through the friction; (iii) Determination of the shape of the sand surface. Since the computational domain is changed due to step (iii), these procedures are repeated until typical shape of the sand dune is formed. Two cases of dunes are simulated. In the first case, when the winds blow from three directions, the sand dune extends at three directions and becomes the shape of a star with three arms. In the second case, when the winds blow from three pairs of opposing directions, the sand dune extends in four directions, becomes the shape of a star with four arms.

1. INTRODUCTION

In the recent years, one of the typical earth-environmental problems on the world is the desertification of more and more tracts of land. There are various reasons for desertification such as excessive herding of domestic animals and excessive farming that leads to the salinization of land. Therefore, we need the advance researches from many fields in order to prevent the desertification. Numerical simulation is one of the fields. If we can know what will happen on the basis of what has happened many years ago, we can find a way to prevent it. This fundamental research is performed in order to predict the movement of the sand dunes.

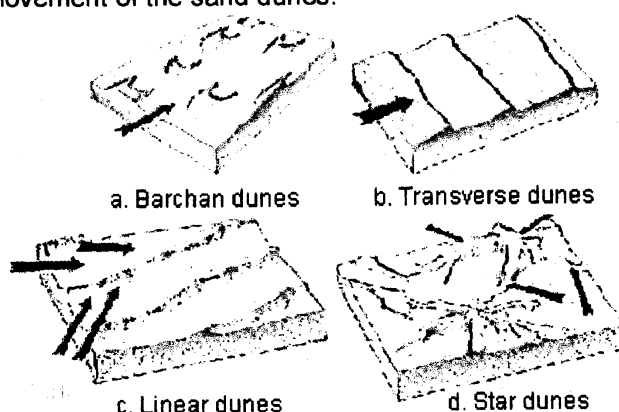


Fig.1 Typical sand dunes²⁾ (Arrows show the wind directions)

Various shapes of sand dunes can be found in the desert. Most dunes can be classified into barchan dunes, transverse dunes, linear dunes and star dunes¹⁾. Barchan dunes and transverse dunes are formed when wind blows from one direction (Fig.1 a and Fig.1 b). The difference between

them is in the sand supply. When sand supply is abundant, transverse dunes are formed, otherwise, barchan dunes are formed. Linear dunes are formed from barchan dunes by two directional winds and extends at the converging direction (Fig.1 c). Star dunes are formed by winds blow from several directions. They commonly have a high central peak and three or more arms extending radially. The arms can vary in length, width, number, and shape, but each has a slip-face (Fig.1 d and Fig.2).

Makiko. Kan simulated the formation of the barchan dunes in 1999³⁾. We simulated the formation of transverse dunes and linear dunes in 2003⁴⁾. In this study, two types of the star dunes are simulated in order to make clear the mechanism of the formation of them. If the flow field over the sand dune is computed, the friction of the wind can be determined. Using the formula between the surface friction and the sand transfer derived by Bagnold, the mass of the sand transfer can be estimated quantitatively. Therefore it is possible to compute the change of the shape and to predict the movement of the sand dunes caused by the wind.

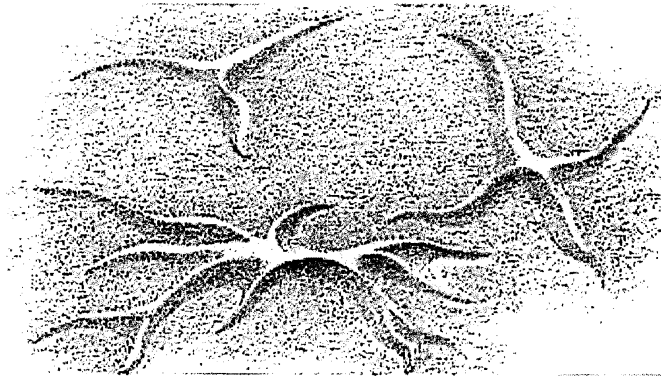


Fig.2 Star dunes²⁾

2. NUMERICAL METHODS

The numerical method employed in this study consists of the following three parts.

- (i) Calculation of the air flow above the sand dunes.
- (ii) Estimation of the sand transfer caused by the air flow through the friction between the air flow and the sand surface.
- (iii) Determination of the shape of the sand dunes.

Because the shape of the sand surface is changed due to step (iii), steps (i)-(iii) are repeated until typical shapes of the sand dunes are formed. We will explain the procedures below.

2.1 Calculation of the Air Flow

Because the strength of the wind in the saltation layer which can transfer the sand is more than 0.02m/s, the Reynolds number of the air flow over the sand surface is high enough that the flow is in turbulence regime. Therefore, we use LES method to compute turbulent flow over the complex geometry. Standard MAC method is employed to solve 3-dimensional Navier-Stokes equations.

The shape of the sand dune is rather complex and is changing with time. In order to impose boundary condition accurately along the sand surface, the time dependent body fitted coordinate system

$$\xi = \xi(x, y, z, t), \quad \eta = \eta(x, y, z, t), \quad \varsigma = \varsigma(x, y, z, t)$$

is used in this study. Then the computation can be done on the time independent rectangular grids⁵⁾.

2.2 Estimation of the Sand Transfer

According to the study of Bagnold⁶⁾, there are three types of sand transfer. They are surface creep, saltation and suspension. These processes are depending on the radius of the sand granularity and the strength of the wind. Observation of the sand transfer shows that saltation is dominant. In this study, we assume that the sand is transferred only by saltation. In the process of saltation, the relation between the sand transfer and the friction velocity u_* is given by⁶⁾

$$q = c \frac{\rho}{g} u_*^3 \quad (1)$$

where u_* is the friction velocity, ρ is the density of the air, q is the mass transfer of the sand. c is experimental constant. The friction velocity is given by

$$|u_*| = \sqrt{\nu \frac{d|U|}{dz}}, \quad u_* \parallel U \quad (2)$$

where U is the velocity parallel to the sand surface and ν is the turbulent eddy viscosity.

2.3 Determination of the Shape of the Sand Dune

The sand dunes are changing its shape by the sand transfer estimated by equation (1). Considering the local coordinate system along the sand surface, continuity equation of sand becomes

$$\rho_s \frac{dh}{dt_h} = -\frac{dq_1}{dX} - \frac{dq_2}{dY} \quad (3)$$

Where ρ_s is the density of the sand and h is the normal distance from the base plane parallel to the sand surface, whereas q_1, q_2 are the X, Y components of the vector q . Axes X and Y are determined by the base plane and the original $x-z$ plane and $y-z$ plane respectively (Fig.3). Eq.3 shows that the increment of h with time equals to the net influx of the sand into the small region. By discretizing Eq.3, the shape of the sand dunes are determined in every time step.

Because the time scale of the change of air flow is quite different from that of the change of the sand surface, the time increment to integrate Eq.3 is 2000 times greater than the one used to integrate the Navier-Stokes equation ($\Delta t_h = 2000\Delta t$). It means that we estimate the sand transfer every $2000\Delta t$.

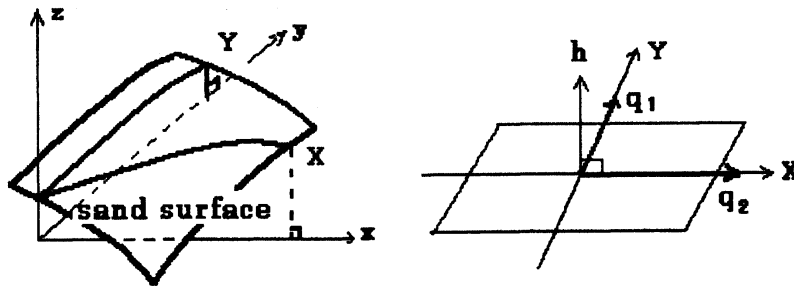


Fig.3 Definition of X and Y in local coordinate system

If the slope of the sand exceeds the maximum angle of about 32° , the height of the sand at the grid point is changed artificially both to keep the maximum value and to satisfy the conservation of the sand. Namely, Δh_1 and Δh_2 are determined by using the relation

$$S_{AX_1OX_1'} = S_{BX_2OX_2'} \quad (\text{Fig.4})$$

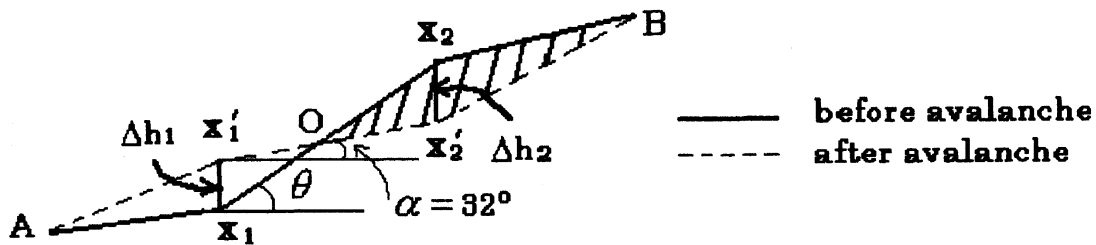


Fig.4 Sand avalanche

3. RESULTS

3.1 Star dunes with three arms

The initial sand dune is shown on Fig.5. A hill with circular cross section parallel to x-y plane and parabolic cross section parallel to both x-z and y-z planes is considered in the simulation analysis. The height of the hill is 25m and the radius of the base is 30m. The number of grid points is 139 in x-direction, 117 in y-direction and 20 in z-direction.

Initial uniform wind is applied in velocity of y-direction with intensity $|u_1| = 8\text{m/s}$ (Fig.5). After 10 hours the wind velocity is changed with $|u_2| = 10\text{m/s}$ and the angle between negative x-axis and wind direction is 60° . After another 10 hours, the wind velocity is changed to $|u_3| = 10\text{m/s}$ and the angle between x-axis and wind direction is 60° . There after, identical set of winds is applied again. It is assumed that the wind blow over the whole region.

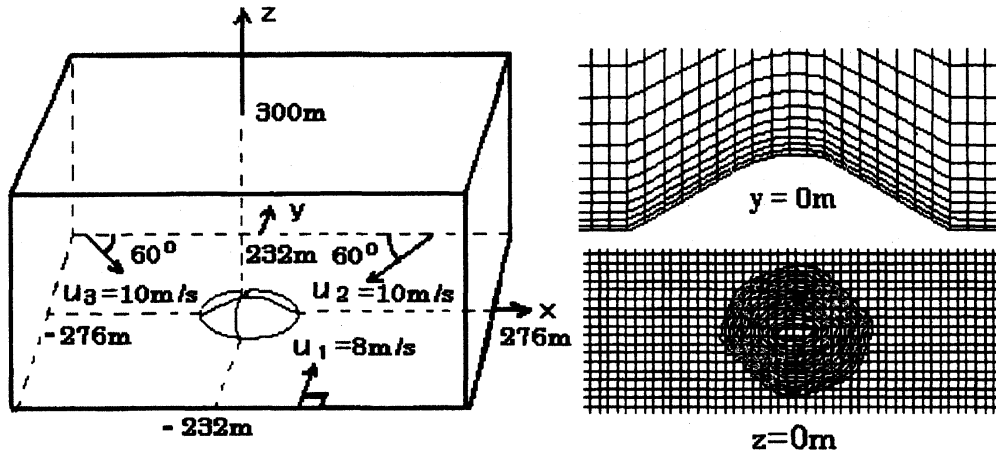


Fig.5 Computational domain (left) and the grid near the hill (right)

The flow field over the fixed sand dune is calculated without movement during the first 1000 steps (20 seconds) in order to obtain the initial conditions. Time increment Δt for the Navier-Stokes equation is set to 0.02s. By using these initial conditions, the steps (i)-(iii) are repeated as mentioned in section 2 and the change of the shape of the sand dune is computed. Although the shape of the sand surface changes with time, no-slip condition is imposed because the sand moves very slowly.

Time development of sand surface contours are presented on Fig.6. When wind is changed every 10 hours, the simulated dune, which has circular cross section parallel to x-y plane and parabolic cross section parallel to both x-z and y-z planes, extends at three directions and becomes the shape of star with three arm. (d means day in Fig. 6)

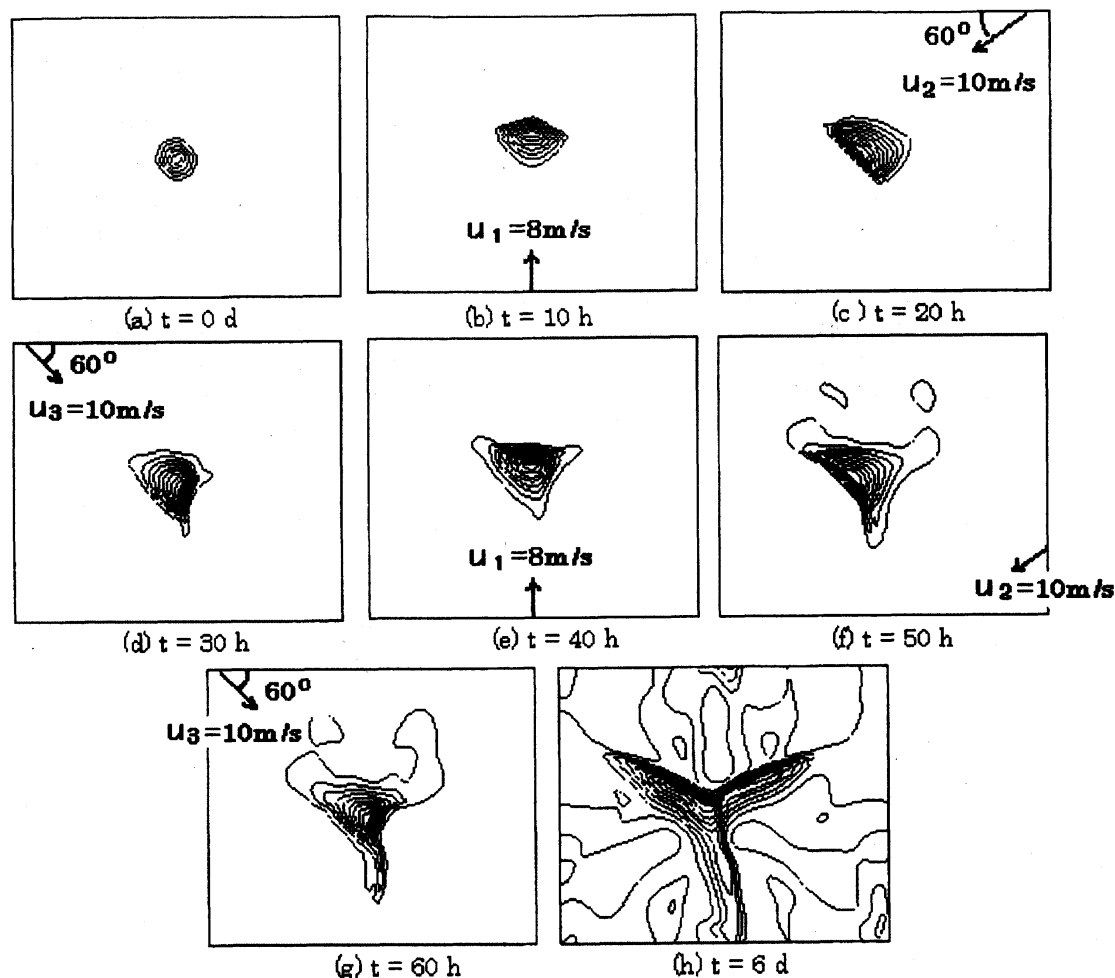


Fig.6 Contours of sand surface

3.2 Star dunes with four arms

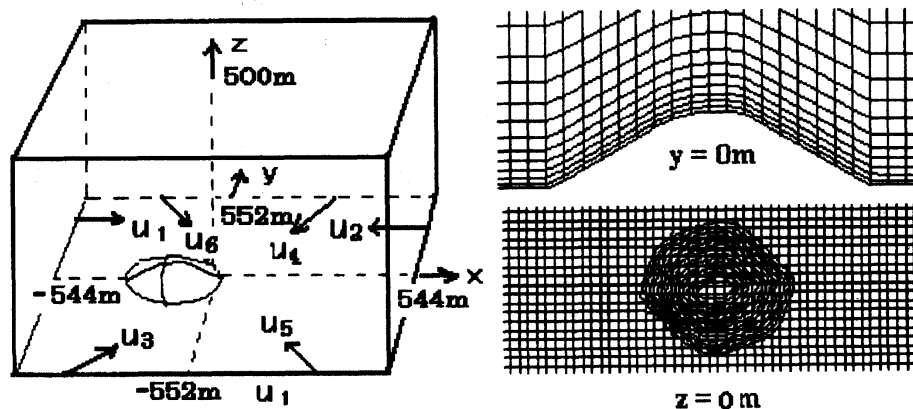


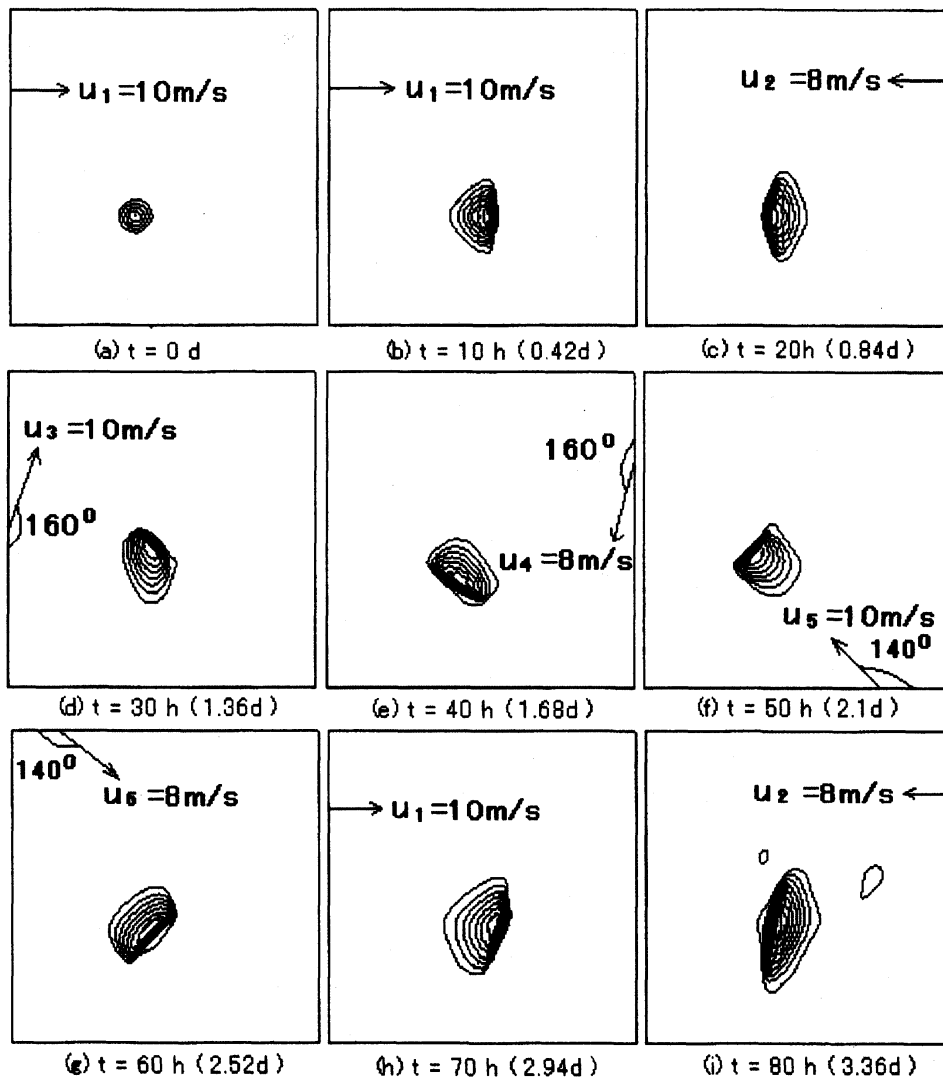
Fig.7 Computational domain (left) and the grid near the hill (right)

The initial sand dune is shown on Fig.7, a hill with circular cross section parallel to x-y plane and parabolic cross section parallel to both x-z and y-z planes. The height of the hill is 20m and the radius of the base is 30m. The number of grid points is 137 in x-direction, 139 in y-direction and 20 in z-direction.

In this simulation, the wind is assumed blow from three pairs of opposing directions. The intensity of the wind is shown on Table 1 and the direction of the wind is shown on Fig.8 (g). Time duration is 10 hours.

As in the previous case, the initial conditions are calculated for the first 1000 steps (20 seconds). The time increment Δt for the Navier-Stokes equation is set to 0.02s. By using these initial conditions, we repeat steps (i)-(iii) as mentioned in section 2 and compute the change of the shape of the sand dune. As the same reason as mentioned in section 3.1, no-slip condition is imposed on the sand surface.

Fig.8 shows the time development of sand surface contours. When winds are blowing form three pairs of opposing directions, the simulated dune, which has circular cross section parallel to x-y plane and parabolic cross section parallel to both x-z and y-z planes, extends at four directions, with high central peak and four arms extending radially.



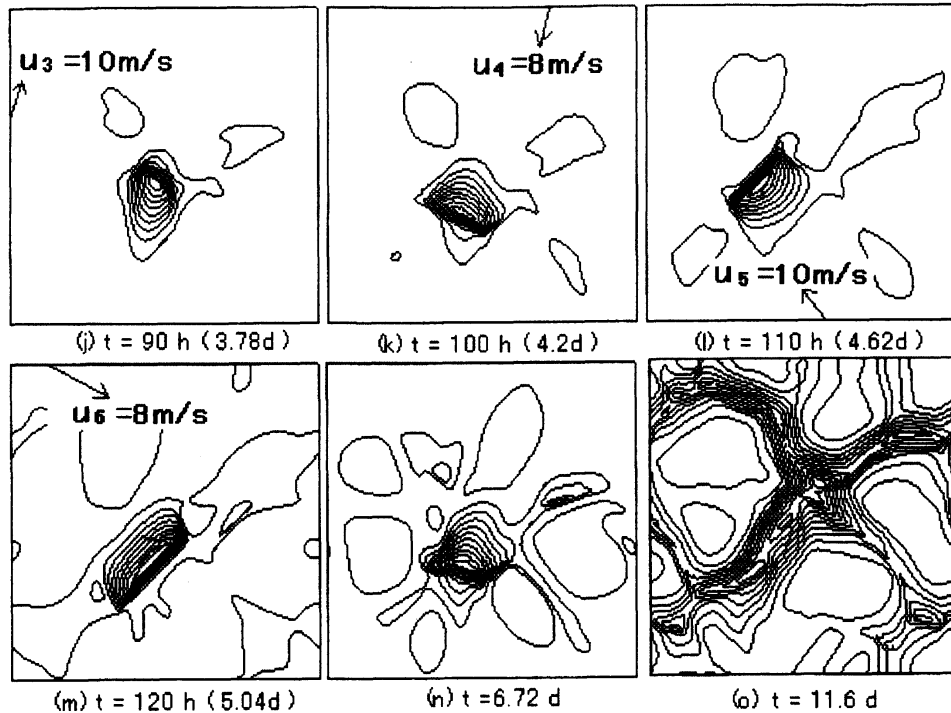


Fig.8 Contours of sand surface

When the sand supply is increased to two times as shown in Fig.9, another shape of star dunes—complex linear dunes are formed. It is explained in our another paper clearly⁷⁾.

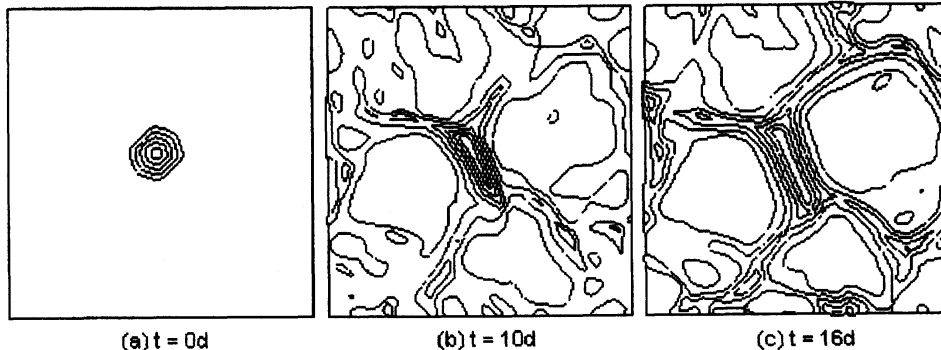


Fig. 9 Contours of sand surface of complex linear dunes

4.CONCLUDING REMARKS

In this study, the formation of star dunes are simulated and the flow above the sand dunes are investigated. One hill is placed on the sand surface as the initial condition.

When the winds blow from three directions, the simulated dune extends at three directions, becoming the shape of a high central peak and three arms extending radially. When the winds blow from three pairs of opposing directions, the simulated dune extends at four directions, becoming the shape of a high central peak and four arms extending radially.

Further problem is to investigate the factors that affect the number of the arms of star dunes and the relationship between the star dunes with four arms and the complex linear dunes.

REFERENCES

- 1) R.A. Wasson and R. Hyde, "Factors determining desert dune type", Nature Vol.304

- (1983), pp.337-339.
- 2) E.D. Mckee, "A study of global sand seas, Introduction to a study of global sand seas", US Geological Survey Professional Paper 1052 (1979), pp.1-19.
 - 3) M. Kan and T. Kawamura, "Numerical simulation of the formation of the barchan sand dune", Theoretical and Applied Mechanics, Vol.48 (1999), pp.349-354.
 - 4) R. Zhang, Y. Sato, M. Kan and T. Kawamura " Numerical study of the effect of flow fields on the shape of sand dune", Theoretical and Applied Mechanic, Vol.52 (2003), pp.205-210.
 - 5) J.F. Thompson, Z.U.A Warsi, C.W Mastin, "Numerical grid generation foundations and applications", Elsevier Science Pubulishing Co. Inc. (1985).
 - 6) R. A. Bagnold, "The movement of desert sand", Proc. Roy. Soc. A157 (1963).
 - 7) Ruyan ZHANG, Makiko KAN and Tetuya KAWAMURA, Numerical Simulation of the Formation of the Complex Linear Dunes, Proceedings of the Sixth World Congress on Computational Mechanics in conjunction with the Second Asian-Pacific Congress on Computational Mechanics, (September 5-10, 2004), Beijing, China, CD.
-